TE- SEM V - EXTC (CBGS) 11/5/16

R.S.A.

QP Code : 31061

(03 Hours) Total Marks: 80

N.B.:

1) **Question Number 1 is Compulsory**
2) Attempt any **Three** questions from the remaining **Five** questions
3) Assumptions made should be clearly stated.
4) Use of normal table is permitted

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1 Answer the following

a) For an LTI system with stochastic input prove that autocorrelation of output is given by convolution of cross-correlation (between input-output) and LTI system impulse response. **05**

b) Suppose that a pair of fair dice are tossed and let the RV X denote the sum of the points. Obtain probability mass function and cumulative distribution function for X. **05**

c) If $Z = X + Y$ and if X and Y are independent then derive pdf of Z as convolution of pdf of X and Y. **05**

d) Write a note on the Markov chains. **05**

2a) Define and Explain moment generating function in detail. **05**

b) Let $Z = X/Y$. Determine $f_Z(z)$ **05**

c) The joint cdf of a bivariate r.v. (X, Y) is given by

$$F_{XY}(x,y) = (1 - e^{-\alpha x})(1 - e^{-\beta y}), x \ge 0, y \ge 0, \alpha, \beta > 0$$
$$= 0 \text{ otherwise.}$$

i) Find the marginal cdf's of X & Y. **02**
ii) Show that X & Y are independent. **02**
iii) Find $P(X\le 1, Y\le 1)$, $P(X\le 1)$, $P(Y>1)$ & $P(X>x, Y>y)$ **06**

3a) Explain strong law of large numbers and weak law of large numbers. **05**

b) Write a note on birth and death queuing models. **05**

c) A distribution with unknown mean μ has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.90 that the sample mean will be within 0.5 of the population mean. **10**

4a) State and prove Chapman-Kolmogorov equation. **05**

b) State and prove Bayes theorem. **05**

c) (i) State any three properties of power spectral density. **03**
(ii) If the spectral density of a WSS process is given by **07**

$$S(w) = b(a-|w|)/a, \quad |w| \le a$$
$$= 0 \quad , \quad |w| > a$$

Find the autocorrelation function of the process.

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5a) The joint probability function of two discrete r.v.'s X and Y is given by $f(x, y) = c(2x + y)$, where x and y can assume all integers such that $0 \le x \le 2$, $0 \le y \le 3$ and $f(x, y) = 0$ otherwise. Find $E(X)$, $E(Y)$, $E(XY)$, $E(X^2)$, $E(Y^2)$, $var(X)$, $var(Y)$, $cov(X, Y)$, and ρ. **10**

b) Prove that if input LTI system is WSS the output is also WSS. What is ergodic process? **10**

6a) The transition probability matrix of Markov Chain is **05**

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 3/4 & 0 & 1/4 \\ 3/4 & 1/4 & 0 \end{bmatrix} \end{array}$$

Find the limiting probabilities.

b) An information source generates symbols at random from a four letter alphabet $\{a, b, c, d\}$ with probabilities $P(a) = 1/2$, $P(b) = 1/4$ and $P(c) = P(d) = 1/8$. A coding scheme encodes these symbols into binary codes as follows: **05**

a	0
b	10
c	110
d	111

Let X be the random variable denoting the length of the code, ie, the number of binary symbols.

i) What is the range of X?

ii) Sketch the cdf $F_X(x)$ of X, and specify the type of X.

iii) Find $P(X \le 1)$, $P(1 < X \le 2)$, $P(X > 1)$ & $P(1 \le X \le 2)$.

c) Write notes on the following: **10**

i) Block diagram and explanation of single & multiple server queuing system

ii) M/M/1/∞ queuing system

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